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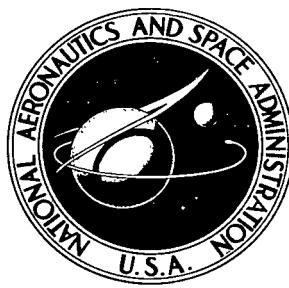
COMPUTER PROGRAM
FOR DIMENSIONAL ANALYSIS

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ABSTRACT

In order to scale models of devices (ships, space capsules, integrated circuits, etc.) and phenomena (nuclear detonations, space-charge widening, sloshing in fuel tanks, etc.), it is necessary to determine nondimensional groupings of variables. A computer program establishes sets of invariant or nondimensional sets of variables and searches for an optimum set under specified optimization criteria. This Fortran IV program is based on algorithms of integer programming. Typical running times are 2 minutes for 10^4 sets of equations.

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SUMMARY

Documentation, illustrative examples, running time, and failure diagnostics are presented for a program useful in scaling models by dimensional analysis. The program, written in FORTRAN IV, performs exclusively integer operations and calculates a basis for the null space of the matrix of a transformation in terms of a given basis. Application to the calculation of B-numbers is made. The program further optimizes a matrix of integers under specified constraints. "Tearing" (or diakoptics) of large systems, defined as minimal under specified constraints, is performed.

The program is available either from the authors or from the Project COSMIC Library of computer programs at the University of Georgia, at Athens.

INTRODUCTION

A Fortran IV program has been developed for the calculation of Buckingham numbers from physical laws. Buckingham numbers are defined as follows. Given a set of variables $[(A_k) \ k = 1,\ldots,n]$, dimensionally expressed in terms of some fixed reference dimensions, a Buckingham number or "B" number is an n-tuple of numbers $[(m_k) \ k = 1,\ldots,n]$, such that the product:

$$\prod_{k=1}^{n} \left(A_k\right)^{m_k}$$

is dimensionless. For example, let the reference dimensions be length, mass, and time (ℓ,m,t), and the variables be E-energy, m-mass, and c-speed of light. Then, these variables dimensionally expressed in terms of the reference dimensions by:

$$E = m\ell^2/t^2$$

$$m = m$$

$$c = \ell/t$$

A "B" number for these variables is $(-1,1,2)$ for $(E)^{-1}(m)^{1}(c)^{2}$ is dimensionless. Thus, "B" numbers express formulas and may be used to find dimensional relationships between variables.

In a system with a large number of variables, more than one "B" number may result. By finding all "B" numbers, it is possible to determine all the ways of decomposing the large system into smaller ones. For instance, if along with the variables E,m,c, we included ℓ and t, in the foregoing example, we could have three "B" numbers $(-1,1,2,00)$ as before; expressing $E = mc^2$, $(-1,1,0,2,-2)$, since the units of E are $m\ell^2/t^2$ and $(0,0,-1,1,-1)$ since the units of c are ℓ/t.

SCOPE

It is assumed that (ℓ_k) and $[(m_k) \; k = 1,...,n]$ are two "B" numbers for the variables (A_k) and t is a real number. Scalar multiplication and addition among "B" numbers are defined by

$$t \times (\ell_k) + (m_k) \equiv (t\ell_k + m_k) \quad k = 1,...,n$$

Observe that:

$$\left(\prod_{k=1}^{n} (A_k)^{t\ell_k + m_k} \right) = \left(\prod_{k=1}^{n} (A_k)^{\ell_k} \right)^{t} \times \left(\prod_{k=1}^{n} (A_k)^{m_k} \right)$$

is dimensionless since (ℓ_k) and (m_k) are "B" numbers. This shows that the "B" numbers form a vector space, with addition and scalar multiplication defined as before.

The program has the following functions.

(1) It determines a basis for the vector space of "B" numbers; that is, it determines a set of "B" numbers such that they are linearly independent, and such that, any other "B" number can be written as a finite combination of the elements of the basis. Such a basis is called a complete set of "B" numbers.

(2) According to criteria previously defined (refs. 1,2,3), the program examines other complete sets of "B" numbers and determines a set in which:

2

(a) The minimal sum of the absolute values of the entries

(b) The maximal number of zero entries

relative to all other complete sets examined.

PROBLEM FORMULATION

Coding the Problem

The input of the problem consists of:

(1) NREFDM = the number of reference dimensions

(2) NVARIB = the number of variables

(3) IVAR = an (NREFDM) by (NVARIB) matrix, which acts as a list (indexed by the second variable) of (NREFDM) by 1 matrices. The J'th entry in this list, or equivalently the J'th column of IVAR, defines the J'th variable in terms of the reference dimensions. For example, if the J'th variable was the Boltzmann constant, which has dimensions (length)(length)(mass)/(time)/(time) (temperature), and if the reference dimensions were (length, mass, time, charge, temperature), then:

$$IVAR(1,J) = 2$$
$$IVAR(2,J) = 1$$
$$IVAR(3,J) = -2$$
$$IVAR(4,J) = 0$$
$$IVAR(5,J) = -1$$

(4) M = NREFDM

(5) N = the number of variables in the formula of interest

(6) IFORM = a list of N numbers which indicate which variables are in the formula of interest. The numbers refer to the second subscript of IVAR.

(7) IOPT = 1 or 2. It indicates which optimization procedure will be followed. 1 means that zeros will be maximized first and the sum of the absolute value of entries will be minimized next, while 2 means that the procedure occurs in the reverse order.

Input Format

The first input card contains NREFDM and NVARIB in 2I5 format. The next NVARIB cards contain NREFDM numbers on each in 14I5 format and form the columns of IVAR. If Q formulas are to be analyzed, then there should be Q sequences of cards in the following form:

> IOPT in I5 format
> M and N in 2I5 format
> IForm (=N numbers) in 14I5 format.

Note that all input data must be in integer form.

Example of Problem Statement (Fig. 1)

The reference dimensions are:

(1) Length
(2) Mass
(3) Time
(4) Charge

The variables are:

(1) Surface area
(2) Mass of electrons
(3) Charge of electrons
(4) Current density
(5) Current
(6) Velocity
(7) Energy
(8) Electric potential.

The 12 input data cards for this problem are shown in Figure 1. The card at the bottom of the page is the first, while the one at the top of the page is the last. The resultant print-out is shown in Figure 2. The program listing is given in the appendix.

The first card indicates that NREFDM = 4, while NVARIB = 8. Each of the next eight cards forms a column of IVAR. The second card, for example, indicated that the dimensions of variable number 1 (= surface area) are $(length)^2$, while the ninth card indicates that the dimensions of variable No. 8 (= electric potential) are $(length)^2 (mass)(time)^{-2}(charge)^{-1}$. The tenth card indicates that IOPT = 1, and the eleventh card that M = 4, N = 8. The last card is IFORM. This card shows which of the variables are to be considered; presently all eight variables are included.

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FORTRAN STATEMENT

IDENTIFICATION

Figure 1.- Input data cards

```
THERE ARE FOUR REFERENCE DIMENSIONS AND EIGHT VARIABLES.
THE ROWS BELOW ARE THE VARIABLES.

        2     0     0     0
        0     1     0     0
        0     0     0     1
       -2     0    -1     1
        0     0    -1     1
        1     0    -1     0
        2     1    -2     0
        2     1    -2    -1


THE FORMULA NOW BEING CONSIDERED IS DEFINED IN TERMS OF THE
FOLLOWING EIGHT VARIABLES.

        1     2     3     4     5     6     7     8


THE COLUMNS OF THE MATRIX BELOW ARE THE VARIABLES IN TERMS OF
THE REFERENCE DIMENSIONS.

        2     0     0    -2     0     1     2     2
        0     1     0     0     0     0     1     1
        0     0     0    -1    -1    -1    -2    -2
        0     0     1     1     1     0     0    -1


THERE ARE FOUR LINEARLY INDEPENDENT B VECTORS.
WE HAVE OPTIMIZED ZEROS FIRST, THEN THE SUM.
THE ROWS OF THE MATRIX BELOW FORM A COMPLETE SET OF B VECTORS.
THE SUM OF THE ABSOLUTE VALUES OF THE ENTIRES IS 19 AND THE
NUMBER OF ZERO ENTRIES IS 19.

       -1     0    -0    -1     1    -0    -0    -0
        0     0     1     0    -0    -0    -1     1
        0    -1    -0     0    -0    -2     1    -0
       -3     0     2    -2     0     2    -0    -0
```

Figure 2.- Computer Print-out for data given by Figure 1.

COMPUTER APPROACH

Complete Sets of "B" Numbers

The program proceeds as follows. Suppose IVAR is a p-by-k matrix, so that each of the k columns of IVAR expresses a variable in terms of the p reference dimension. A k-tuple $(n_1,\ldots,n_k)$ of numbers is a "B" number if and only if

$$0 = \text{IVAR} \cdot \begin{bmatrix} n_1 \\ \vdots \\ n_k \end{bmatrix}$$

and this equals the matrix product of IVAR and the column vector

$$\begin{bmatrix} n_1 \\ \vdots \\ n_k \end{bmatrix}.$$

If we consider IVAR as a linear transformation, then the vector space of "B" numbers is exactly equal to the null space of IVAR, and in order to find a complete set of "B" numbers, it suffices to find a basis for this null space. This is accomplished next.

Let $e_i = (0,\ldots,0,1,0,\ldots,0)$ be a k-tuple where the 1 occurs in the i-th spot, $i = 1,\ldots,k$. Form the matrix whose i-th row consists of IVAR (e_i). The matrix is k by p. Row reduce this matrix. As a by-product of the row reduction process one obtains a permutation π of $(1,\ldots,k)$ and number a_{ig} such that $a_{ii} \neq 0$ and such that the rows of the row reduced matrix are given by

$$\text{IVAR}[e_{\pi(1)}]$$

$$a_{21}\text{IVAR}[e_{\pi(1)}] + a_{22}\text{IVAR}[e_{\pi(2)}]$$

$$a_{31}\text{IVAR}[e_{\pi(1)}] + a_{32}\text{IVAR}[e_{\pi(2)}] + a_{33}\text{IVAR}[e_{\pi(3)}]$$

$$\ldots$$

$$a_{k1}\text{IVAR}[e_{\pi(1)}] + \ldots + a_{kk}\text{IVAR}[e_{\pi(k)}]$$

where the first s rows are non-zero and the last k-s rows are zero. Let

$$w_1 = e_{\pi(1)}$$

and let

$$w_i = \sum_{y=1}^{i} a_{iy} e_{\pi(y)}. \quad i = z,\ldots,k.$$

Since the e_i's are linearly independent and since $a_{ii} \neq 0$, it follows that the w_i's are linearly independent. Let W_1 be the space spanned by $w_1,\ldots,w_s$ and let W_2 be the space spanned by $w_{s+1},\ldots,w_k$. IVAN $(W_1 \oplus W_2)$ = IVAR (W_1) so that W_2 is the null-space of IVAR. Let θ be the inverse permutation to π. Then

$$[a_{i\ \theta(y)}] \quad i = 1 + 1,\ldots,k$$

expresses a basis of W_2 in terms of the original one. (Note that the e_i could have been chosen to be any k linearly independent k-tuples.)

In this way the program determines a complete set of "B" numbers. The input entries of IVAR must be integers, and as a result, the entries of the "B" vectors forming a complete set are integer.

Optimization

Next, it is determined if there are any common factors among the entries of the "B" numbers, and whether there is only one "B" number in the complete set. If there is, there is no need to go through the optimizing procedure. But if there is more than one "B" number, then an attempt to find a more optimal complete set is made. The procedure is as follows.

Suppose $V_1,\ldots,V_k$ is the complete set of "B" numbers which we have. If

$$W_1 = A_1 V_1 + \ldots + A_k V_k \text{ where } A_1,\ldots,A_k \text{ are integers and}$$

$$A_1 \neq 0, \text{ then } W_1, V_2,\ldots,V_k \text{ is also a complete set.}$$

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In this program, $A_1,...,A_k$ takes on the values $-2,-1,0,1,2$ and the different W_1's so obtained are compared and an optimal one is chosen. Then, $V_1 = V_k$, $V_2 = W_1$, $V_3 = V_2,...,$ $V_k = V_{k-1}$ are set. This process is repeated k-times. There are at present two choices of the optimizing procedure. The first choice is to maximize the number of zero entries and then minimize the sum of the absolute value, while the second is just the reverse. The two procedures do not appear to be equivalent.

After performing the foregoing operations on one formula, the program goes on and reads another formula.

Performance

The program has been tested on an IBM 7094. The running time for 28 formulas, each with 1,2,3, or 4 "B" numbers in its complete set, was about 2 minutes. The time for a formula with 5 "B" numbers in its complete set was slightly more than 4 minutes. This time includes an assembly listing and the punching of a binary deck. Without the listing and deck, a Univac 1108 took 42 seconds to do the same 28 formulas.

Electronics Research Center
  National Aeronautics and Space Administration
    Cambridge, Massachusetts, September 1968
      129-06-02-75

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APPENDIX

PROGRAM LISTING

MAIN   — EFN   SOURCE STATEMENT   —   IFN(S)   —

```
C
C          THIS PROGRAM PERFORMS THE FOLLOWING FUNCTION.  GIVEN A
C     SET OF REFERENCE DIMENSIONS, A SET OF VARIABLES EXPRESSED
C     IN TERMS OF THOSE DIMENSIONS AND A FORMULA IN TERMS OF SOME
C     OF THE VARIABLES, THE PROGRAM FIRST DETERMINES A COMPLETE
C     (I.E.,MAXIMAL LINEARLY INDEPENDENT) SET OF B-VECTORS.  THEN
C     IT EXAMINES OTHER COMPLETE SETS OF B-VECTORS AND DETERMINES
C     WHICH HAS THE MINIMAL SUM OF THE  ABSOLUTE VALUES OF THE
C     ENTRIES AND WHICH HAS THE MAXIMAL NUMBER OF ZEROS IN THE
C     ENTRIES OF THE B-VECTORS.
C
C          THE INPUT CONSISTS OF
C     1.   NREFDM = THE NUMBER OF REFERENCE DIMENSIONS.
C     2.   NVARIB = THE NUMBER OF VARIABLES
C     3.  IVAR = AN (NREFDM) BY (NVARIB) MATRIX WHICH ACTS AS A
C     LIST(INDEXED BY THE SECOND VARIABLE) OF (NREFDM) BY 1
C     MATRICES. THE J'TH ENTRY IN THIS LIST , OR EQUIVALENTLY
C     THE J'TH COLUMN OF IVAR DEFINES THE J'TH VARIABLE IN TERMS
C     OF THE REFERENCE DIMENSIONS.  FOR EXAMPLE IF THE J'TH
C     VARIABLE WAS THE BOLTZMANN CONSTANT WHICH HAS DIMENSIONS
C     (LENGTH)(LENGTH)(MASS)/(TIME)(TIME)(TEMPERATURE)
C     AND IF THE REFERENCE DIMENSIONS WERE
C     (LENGTH,MASS,TIME,CHARGE,TEMPERATURE)
C     THEN
C               IVAR(1,J)=2
C               IVAR(2,J)=1
C               IVAR(3,J)=-2
C               IVAR(4,J)=0
C               IVAR(5,J)=-1.
C     4.  M=NREFDM
C     5.  N= THE NUMBER OF VARIABLES IN THE FORMULA OF INTEREST.
C     6.  IFORM= A LIST OF N NUMBERS WHICH INDICATE WHICH
C     VARIABLES ARE IN THE FORMULA OF INTEREST. THE NUMBERS REFER
C     TO THE SECOND SUBSCRIPT OF IVAR.
C     7.  IOPT= 1 OR 2. IT INDICATES WHICH OPTIMIZATION PROCEDURE
C     WILL BE FOLLOWED.  1 MEANS THAT ZEROS WILL BE MAXIMIZED
C     FIRST AND THE SUM OF THE ABSOLUTE VALUE OF THE ENTRIES WILL
C     BE MINIMIZED NEXT,WHILE 2 MEANS THAT THE PROCEDURE OCCURS
C     IN THE REVERSE ORDER.
C
C          INPUT FORMAT
C          THE FIRST INPUT CARD CONTAINS NREFDM AND NVARIB IN
C     215 FORMAT.  THE NEXT NVARIB CARDS CONTAIN NREFDM NUMBERS
C     ON EACH IN 14I5 FORMAT AND FORM THE COLUMNS OF IVAR.
```

```
C              IF Q FORMULAS ARE TO BE ANALYZED, THEN THERE SHOULD BE
C              Q SEQUENCES OF CARDS IN THE FOLLOWING FORM...
C
C
C              IOPT IN I5 FORMAT.
C              M AND N IN 2I5 FORMAT
C              IFORM(=N NUMBERS) IN 14I5 FORMAT
C
C
      DIMENSION MULT(30),NULSAV(30,30),MATSZ(30,30),INMAT(30,30)
      DIMENSION IV(15,15),NULL(30,30),IVAR(5,40),IFORM(15),MATSS(30,30)
  101 FORMAT(2I5)
  102 FORMAT(14I5)
 7000 FORMAT(10H THERE ARE,I2,25H REFERENCE DIMENSIONS AND,I2,11H VARIAB
     1LES.)
 7001 FORMAT(34H THE ROWS BELOW ARE THE VARIABLES.)
 7002 FORMAT(////70H THE FORMULA NOW BEING CONSIDERED IS DEFINED IN TERM
     1S OF THE FOLLOWING,I2,11H VARIABLES./)
 7004 FORMAT(/88H THE COLUMNS OF THE MATRIX BELOW ARE THE VARIABLES IN T
     1ERMS OF THE REFERENCE DIMENSIONS.)
      READ (5,101) NREFDM,NVARIB
      WRITE (6,7000) NREFDM,NVARIB
      WRITE (6,7001)
      DO 1900 J=1,NVARIB
      READ (5,102) (IVAR(I,J),I=1,NREFDM)
 1900 WRITE (6,102) (IVAR(I,J),I=1,NREFDM)
  130 FORMAT(I5)
  100 READ (5,130) IOPT
      READ (5,101) M,N
      WRITE (6,7002) N
      READ (5,102) (IFORM(J),J=1,N)
      WRITE (6,102) (IFORM(J),J=1,N)
      DO 111 J=1,N
      IGN=IFORM(J)
      DO 111 I=1,M
  111 INMAT(I,J)=IVAR(I,IGN)
      WRITE (6,7004)
      DO 7003 I=1,M
 7003 WRITE (6,102) (INMAT(I,J),J=1,N)
      DO 1 I1=1,N
      DO 1 J1=1,N
    1 IV(I1,J1)=0
      DO 2 I2=1,N
    2 IV(I2,I2)=1
C
C
C              FIND A BASIS FOR THE NULL SPACE
C
    3 CALL NULSPA(M,N,INMAT,IV,NULL,K)
C
C              THE ROWS OF NULL ARE B VECTORS AND FORM A COMPLETE SET.
C
C              K IS THE NUMBER OF B VECTORS IN A COMPLETE SET.
C
C
C              ISUMS IS THE SUM OF THE ABSOLUTE VALUE OF THE ENTRIES IN
```

```
C                    THE FINAL OPTIMIZED MATRIX,AND IZERS IS THE NUMBER OF ZERO!
C
      ISUMS=0
      IZERS=0
C
C            ARE THERE ANY NON-ZERO B VECTORS
C
      IF(K-1)200,620,801
  200 WRITE (6,103)
  103 FORMAT(38H THE ONLY B VECTOR IS THE ZERO VECTOR.)
      GO TO 100
C
C            IF THERE IS ONLY ONE B VECTOR THERE IS NO NEED TO GO
C            THROUGH THE OPTIMIZATION ROUTINE.
C
  620 IZERS=0
      ISUMS=0
C
C            CHECK TO SEE IF ANY OF THE NUMBERS 2,3,5,7,11 DIVIDES
C            EVERY ENTRY OF A B VECTOR.
C
      CALL DIVCHK(1,N,NULL)
  902 DO 622 J=1,N
      ISUMS=ISUMS+IABS(NULL(1,J))
      MATSZ(1,J)=NULL(1,J)
      IF(NULL(1,J))622,621,622
  621 IZERS=IZERS+1
  622 CONTINUE
      GO TO 307
C
C            SAVE ORIGINAL COMPLETE SET OF B VECTORS IN MATRIX NULSAV.
C
  801 KK=K-1
      DO 9000 IXD=1,K
      DO 9000 JXD=1,N
 9000 NULSAV(IXD,JXD)=NULL(IXD,JXD)
C
C            NOW GO THROUGH OPTIMIZATION.
C
C
C            WE HAVE A COMPLETE(I.E.,MAXIMAL LINEARLY INDEPENDENT)
C            SET OF B VECTORS,SAY  V(1),V(2),...,V(K).
C            IF    W(1)=A*V(1)+B*V(2)+...+C*V(K),WHERE A,B,...,C ARE
C            INTEGERS AND A IS NOT ZERO,THEN W(1),V(2),...,V(K) IS
C            AGAIN A COMPLETE SET OF B VECTORS.   IN THIS PROGRAM
C            A,B,...,C TAKE ON THE VALUES -2,-1,0,1,2 AND THE DIFFERENT
C            W(1)'S ARE   OBTAINED IN THIS WAY.  THEY ARE COMPARED AND
C            AN OPTIMAL ONE IS CHOSEN.   THEN WE SET V(1)=V(K),V(2)=W(1),
C            V(3)=V(2),...,V(K)=V(K-1).   THIS PROCESS IS REPEATED K
C            TIMES.
C
C
C            MAJROW INDICATES HOW MANY TIMES WE HAVE GONE THROUGH THE
C            ABOVE PROCESS.
C
      DO 3014 MAJROW=1,K
```

```
C
C              ISUMSA IS THE SUM OF THE ABSOLUTE VALUES OF THE ENTRIES IN
C              THE MOST OPTIMAL B VECTOR YET EXAMINED,AND IZERSA IS THE
C              NUMBER OF ZEROS.
C
       ISUMSA=9999
       IZERSA=-1
       IUP=2**KK
C
C              NOW WE ARE DETERMINING THE A,B,...,C,WHICH WE CALL JZEROX,
C              MULT(1),...,MULT(K-1),RESPECTIVELY.
C
       DO 670 IZEROX=1,5
       IF(IZEROX-3)650,670,650
  650  JZEROX=IZEROX-3
       DO 3006 INUMB1=1,IUP
       INUMB1=INUMB1-1
       DO 3006 INUMB2=1,IUP
       INUMB2=INUMB2-1
       DO 3006 INUMB3=1,IUP
       INUMB3=INUMB3-1
       DO 350 JNUMB=1,KK
       ITEST=2**(JNUMB-1)
       MULT(JNUMB)=0
       IF(AND(INUMB1,ITEST).NE.0.) GO TO 612
       GO TO 350
  612  MULT(JNUMB)=1
       IF(AND(INUMB2,ITEST).NE.0.) MULT(JNUMB)=-MULT(JNUMB)
       IF(AND(INUMB3,ITEST).NE.0.) MULT(JNUMB)=2*MULT(JNUMB)
  350  CONTINUE
       DO 351 J=1,N
  351  NULL(MAJROW,J)=JZEROX*NULSAV(1,J)
C
C              NULL(MAJROW,.) IS THE NEW B VECTOR WHICH WE WILL EXAMINE.
C
       DO 354 I=2,K
       I1=I-1
  352  DO 353 J=1,N
  353  NULL(MAJROW,J)=NULL(MAJROW,J)+(MULT(I1)*NULSAV(I,J))
  354  CONTINUE
C
C              CHECK TO SEE IF ANY OF THE NUMBERS 2,3,5,7,11 DIVIDES
C              EVERY ENTRY OF A B VECTOR.
C
     7 CALL DIVCHK(MAJROW,N,NULL)
C
C              CHECK OPTIMIZING CONDITIONS.  DEPENDING ON IOPT,EITHER
C              FIRST DETERMINE THE NUMBER OF ZERO ENTIRES AND THEN SUM THE
C              ABSOLUTE VALUES OF THE ENTRIES OR VICE VERSA.
C
  802  IZERO=0
       ISUM=0
       DO 9 J5=1,N
       ISUM=ISUM+IABS(NULL(MAJROW,J5))
       IF(NULL(MAJROW,J5))9,8,9
     8 IZERO=IZERO+1
```

```
      9 CONTINUE
        GO TO (723,724),IOPT
    723 IF(IZERO-IZERSA)3006,6001,3003
   6001 IF(ISUM-ISUMSA)3003,3006,3006
    724 IF(ISUM-ISUMSA)3003,6002,3006
   6002 IF(IZERO-IZERSA)3006,3006,3003
   3003 IZERSA=IZERO
        ISUMSA=ISUM
        DO 3004 IK4=1,N
   3004 MATSZ(MAJROW,IK4)=NULL(MAJROW,IK4)
C
C              MATSZ IS THE MOST OPTIMAL NEW B VECTOR YET EXAMINED.
C
   3006 CONTINUE
    670 CONTINUE
        DO 760 J=1,N
    760 NULSAV(1,J)=NULSAV(K,J)
        DO 761 I=2,K
        DO 761 J=1,N
    761 MATSS(I,J)=NULSAV(I,J)
        DO 762 I=2,KK
        DO 762 J=1,N
        I2=I+1
    762 NULSAV(I2,J)=MATSS(I,J)
        DO 763 J=1,N
    763 NULSAV(2,J)=MATSZ(MAJROW,J)
        ISUMS=ISUMS+ISUMSA
        IZERS=IZERS+IZERSA
   3014 CONTINUE
C
C
C              WE ARE DONE TESTING, PRINT ANSWER
C
    307 WRITE (6,104) K
    104 FORMAT(10H THERE ARE,I3,32H LINEARLY INDEPENDENT B VECTORS.)
        GO TO (131,132),IOPT
    131 WRITE (6,133)
    133 FORMAT(44H WE HAVE OPTIMIZED ZEROS FIRST,THEN THE SUM.)
        GO TO 4006
    132 WRITE (6,134)
    134 FORMAT(44H WE HAVE OPTIMIZED THE SUM FIRST,THEN ZEROS.)
   4006 WRITE (6,107)
    107 FORMAT(63H THE ROWS OF THE MATRIX BELOW FORM A COMPLETE SET OF B V
       1ECTORS.)
        WRITE (6,120) ISUMS,IZERS
    120 FORMAT(49H THE SUM OF THE ABSOLUTE VALUES OF THE ENTRIES IS,I3,34H
       1 AND THE NUMBER OF ZERO ENTRIES IS,I3)
        DO 315 I9=1,K
    315 WRITE (6,106) (MATSZ(I9,J9),J9=1,N)
    106 FORMAT(15I5)
        GO TO 100
        END
```

15

```
      SUBROUTINE BASE(M,N,INMAT,MATOUT,K)
      DIMENSION INMAT(30,30),MATOUT(30,30),ICONS2(15),MATSAV(30,30)
C              SUBROUTINE BASE HAS THE FOLLOWING FUNCTION. GIVEN A VECTOR
C     SPACE V WITH BASIS(V1,V2,...,VN) AND A SET OF VECTORS IN V (W1,W2,
C     ...,WM),BASE DETERMINES-1.K=THE DIMENSION OF W,WHERE W IS THE
C     SUBSPACE GENERATED BY THE WI'S,AND
C                              -2.K VECTORS IN W,IN TERMS OF THE ORIGINAL
C     WI'S, WHICH ARE A BASIS FOR W.
C          THE INPUT CONSISTS OF- 1.N=THE DIMENSION OF V
C                                 2.M=THE NUMBER OF VECTORS IN THE GIVEN
C     SET,AND
C                                 3.INMAT=THE M BY N MATRIX WHICH
C     EXPRESSES THE WI'S IN TERMS OF THE VJ'S.
C          THE OUTPUT CONSISTS OF- 1.K=THE DIMENSION OF W,AND
C                                  2.MATOUT=THE M BY M MATRIX WHOSE
C     FIRST K ROWS DETERMINES THE BASIS OF W IN TERMS OF THE VECTORS IN
C     THE GIVEN SET.
C          NOTE-THE INPUT MATRIX MUST HAVE INTEGRAL ENTRIES.
C
      DO 61 I=1,M
      DO 61 J=1,M
      MATSAV(I,J)=INMAT(I,J)
      IF(I-J)1,60,1
    1 MATOUT(I,J)=0
      GO TO 61
   60 MATOUT(I,J)=1
   61 CONTINUE
      IROW=1
C              IROW INDICATES WHICH ROW WE ARE NOW AT IN THE ROW REDUCTION
C              PROCESS. IT ALSO INDICATES HOW MANY VECTORS IN THE BASIS
C              HAVE ALREADY BEEN CONSTRUCTED.
C
C              FIND A NON-ZERO VECTOR
C
      DO 50 J=1,N
      DO 50 I=1,M
      IF(INMAT(I,J))51,50,51
   51 JSTART=J
      IF(I-M)30,33,33
   30 ISTART=I
      GO TO 31
   50 CONTINUE
      K=0
      RETURN
   31 IPLUS=ISTART+1
      DO 32 I=IPLUS,M
      DO 32 J=1,N
      IF(INMAT(I,J))52,32,52
   32 CONTINUE
   33 K=1
      RETURN
   52 DO 15 J=JSTART,N
      DO 3  I=IROW,M
      IF (INMAT(I,J))2,3,2
    2 ISWOP=I
```

```
          GO TO 4
      3 CONTINUE
          GO TO 15
C
C               INTERCHANGE ROWS OF INMAT.
C
      4 DO 5 J1=1,N
          SAVE=INMAT(IROW,J1)
          INMAT(IROW,J1)=INMAT(ISWOP,J1)
      5 INMAT(ISWOP,J1)=SAVE
C
C               INTERCHANGE CORRESPONDING ROWS OF MATOUT.
          DO 6 I1=1,M
          SAVE=MATOUT(IROW,I1)
          MATOUT(IROW,I1)=MATOUT(ISWOP,I1)
      6 MATOUT(ISWOP,I1)=SAVE
      9 IROW1=IROW+1
          ICONS1=INMAT(IROW,J)
C
C               ROW REDUCE INMAT.
C       CHANGE MATOUT ACCORDINGLY.
C
          DO 66 I3=IROW1,M
          ICONS2(I3)=INMAT(I3,J)
          IF(ICONS2(I3))65,66,65
     65 DO 10 J3=1,N
     10 INMAT(I3,J3)=(ICONS1*INMAT(I3,J3))-(ICONS2(I3)*INMAT(IROW,J3))
     66 CONTINUE
          DO 68 I4=IROW1,M
          IF(ICONS2(I4))67,68,67
     67 DO 13 J4=1,M
     13 MATOUT(I4,J4)=(ICONS1*MATOUT(I4,J4))-(ICONS2(I4)*MATOUT(IROW,J4))
     68 CONTINUE
          IF(J-N)40,19,19
     40 IF(IROW1-M)41,16,19
     41 JONE=J+1
          DO 42 JTRY=JONE,N
          DO 42 ITRY=IROW1,M
          IF(INMAT(ITRY,JTRY))14,42,14
     42 CONTINUE
          GO TO 19
     14 IROW=IROW+I
     15 CONTINUE
          GO TO 19
     16 DO 17 I5=1,N
          IF(INMAT(IROW1,I5))18,17,18
     17 CONTINUE
     19 K=IROW
          GO TO 71
     18 K=IROW1
     71 DO 70 I6=1,M
          DO 70 I7=1,N
     70 INMAT(I6,I7)=MATSAV(I6,I7)
          RETURN
          END
```

17

```
      SUBROUTINE NULSPA(M,N,L,IV,NULL,K)
C
C          WE ARE GIVEN TWO VECTOR SPACES V AND W WITH BASIS V(1),...
C     V(N) AND W(1),...,W(M),RESPECTIVELY, AND A LINEAR TRANSFORMA-
C     TION L FROM V INTO W. SUBROUTINE NULSPA DETERMINES A BASIS FOR
C     THE NULL SPACE OF L IN TERMS OF A GIVEN BASIS V.
C          V(1),...,V(N)  IS THE USUAL BASIS, NAMELY V(I)=(0,...,0,
C     1,0,...,0) WHERE THE 1 OCCURS IN THE I'TH LOCATION.
C
C     THE INPUT CONSISTS OF-
C
C     1. TWO INTEGERS M AND N.
C
C     2.AN N BY N MATRIX IV WHOSE COLUMNS ARE A BASIS OF V, AND
C
C     3.AN M BY N MATRIX WHICH CONRRESPONDS TO L IN THE GIVEN BASIS.
C
C
C     THE OUTPUT CONSISTS OF-
C
C     1.K = THE DIMENSION OF THE NULL SPACE OF L, AND
C
C     2.NULL = A K BY N MATRIX WHOSE J'TH ROW (J1,...JN) REPRESENTS
C          THE J'TH VECTOR = J1*V(1) +...+JN*V(N) OF THE BASIS OF THE
C          NULL SPACE.
C
C
C          THE METHOD.
C          LET V(I) BE THE I'TH COLUMN VECTOR OF IV.  STEP 1
C     CONSISTS OF FORMING THE MATRIX IRANGE WHOSE I'TH ROW CONSIS
C     TS OF L(V(I)).  STEP 2 CONSISTS OF CALLING SUBROUTINE BASE
C     WHICH ROW REDUCES IRANGE.  IT ALSO DETERMINES THE ROWS OF
C     THE ROW REDUCED MATRIX INTERMS OF THE INITIAL ROWS OF THE
C     MATRIX  AND EXPRESSES THE RESULT IN THE MATRIX  IBSRAN.
C          IDMRAN IS THE DIMENSION OF THE RANGE OF L.  THE
C     FIRST IDMRAN ROWS OF IBSRAN FORM A BASIS OF THE RANGE OF L
C     AS FOLLOWS.
C          WE HAVE V(1),...,V(N) AS A BASIS OF V AND WE FORMED
C     L(V(I)) FOR I=1,...,N.  SUPPOSE IBSRAN=(A(I,J)).  THEN THE
C     I'TH ELEMENT OF A BASIS FOR THE RANGE OF L IS GIVEN BY
C     A(I,1)*L(V(1))+...+A(I,N)*L(V(N)).  SINCE L IS LINEAR IT
C     FOLLOWS THAT THE VECTORS   A(I,1)*L(V(1)+...+A(I,N)*V(N),
C     FOR I=1,...,IDMRAN,FORM A BASIS FOR A SUBSPACE OF V WHICH IS
C     ISOMORPHIC, UNDER THE RESTRICTION OF L TO THIS SUBSPACE,
C     TO THE RANGE OF L.  THE LAST  N-IDMRAN  ROWS OF THE ROW
C     REDUCED IRANGE MATRIX ARE ALL ZERO.  SO A(I,1)*L(V(1))+...+
C     A(I,N)*L(V(N))=0,FOR I=IDMRAN+1,...,N.  THE LINEARITY OF L
C     SHOWS THAT THE VECTORS  A(I,1)*V(1)+...+A(I,N)*V(N),FOR I=
C     IDMRAN+1,...,N, ARE IN THE NULL SPACE OF L.  THE ROW
C     REDUCTION PROCESS SHOWS THAT THESE VECTORS ARE LINEARLY
C     INDEPENDENT.  HENCE THE LAST N-IDMRAN ROWS OF IBSRAN ARE A
C     BASIS FOR THE NULL SPACE OF L.
C
      DIMENSION L(30,30),NULL(30,30)
      DIMENSION IV(15,15)
```

```
      DIMENSION IRANGE(30,30),IBSRAN(30,30)
      DO 1 I1=1,M
      DO 1 J1=1,N
    1 IRANGE(J1,I1)=0
C
C           STEP 1
      DO 7 I2=1,M
      DO 7 J2=1,N
      DO 7 J3=1,N
    7 IRANGE(J2,I2)=IRANGE(J2,I2)+(L(I2,J3)*IV(J3,J2))
C
C           STEP 2
      CALL BASE(N,M,IRANGE,IBSRAN,IDMRAN)
      K=N-IDMRAN
      DO 6 I2=1,K
      I3=IDMRAN+I2
      DO 6 J2=1,N
    6 NULL(I2,J2)=IBSRAN(I3,J2)
      RETURN
      END
```

19

```
      SUBROUTINE DIVCHK(L2,N,NULL)
      DIMENSION NULL(30,30)
C
C     DIVCHK CHECKS TO SEE IF THE  NUMBERS 2,3,5,7,11 DIVIDE EVERY ENTRY
C     OF ROW L2 IN THE MATRIX NULL WHICH HAS  N  COLUMNS.  IF SO  THEN
C     THE DIVISION IS DONE.
C
      ICNT=1
  732 GO TO (726,727,728,729,730),ICNT
  726 L3=2
      GO TO 731
  727 L3=3
      GO TO 731
  728 L3=5
      GO TO 731
  729 L3=7
      GO TO 731
  730 L3=11
  731 DO 720 L4=1,N
      ICHECK=MOD(NULL(L2,L4),L3)
      IF(ICHECK)722,720,722
  720 CONTINUE
      DO 721 L5=1,N
  721 NULL(L2,L5)=NULL(L2,L5)/L3
      GO TO 732
  722 IF(ICNT-5)733,802,802
  733 ICNT=ICNT+1
      GO TO 732
  802 RETURN
      END
```

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